Visualizing and Interpreting ODEs

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Cancer Systems Biology

Review

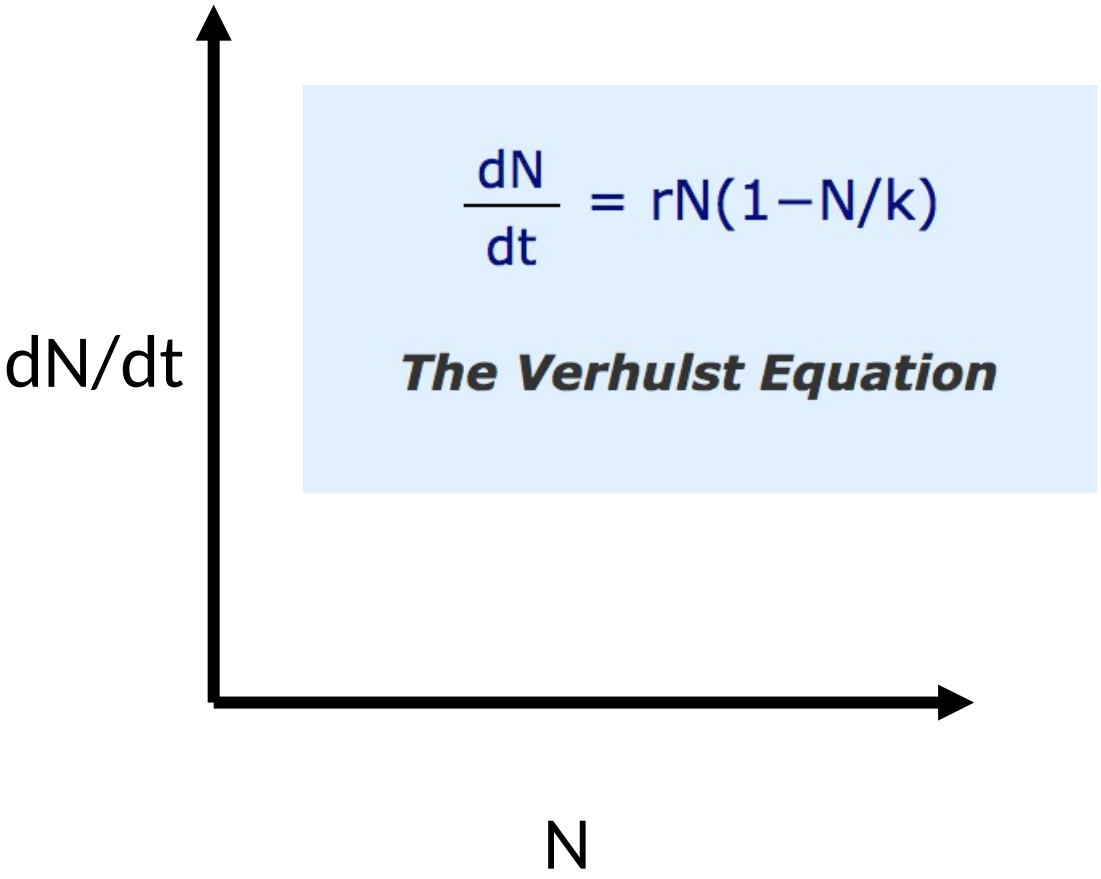
- What was the population dynamics model? (logistic model?)
  - If we use the definition of differential equations we talked about last class, how would you interpret this ODE?

dN/dt = rate of change of population

$N(t)$ = population size at time t

Solve the equation

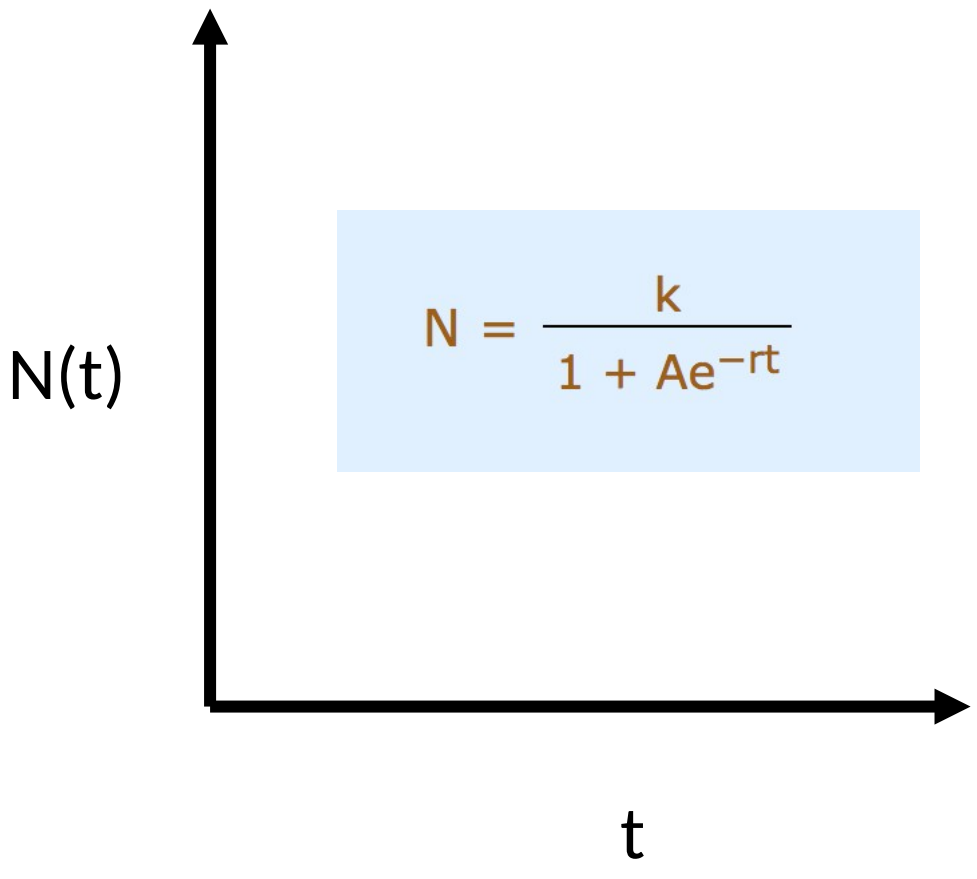
What does the **ODE** (function of N) look like on this graph?



$$\frac{dN}{dt} = rN(1-N/k)$$

The Verhulst Equation

What does the **solution** (**N(t)**) to the ODE look like on this graph?



$$N = \frac{k}{1 + Ae^{-rt}}$$

Interpreting the logistic growth model

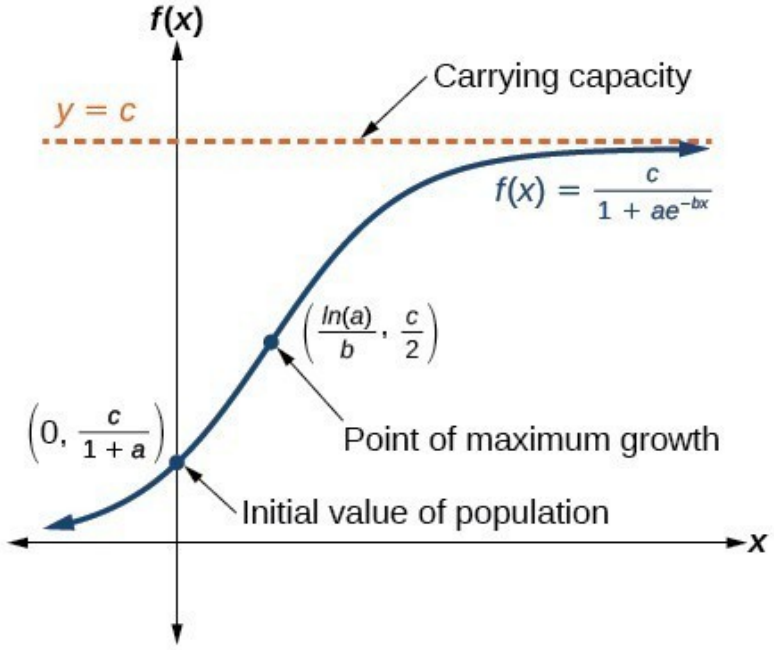
- What does r represent? Explain how a certain value of r affects the differential equation. What about in the solution? (What happens if r is big versus small)
- Is the rate of change in N going to increase or decrease over time?
- What is k? Explain how a certain value of K affects the differential equation. What about in the solution? (What happens if K is big versus small?)
- What is the relationship between the population size and the rate of change of the population size? What does this tell you about the dynamics of the population?

$$\frac{dN}{dt} = rN(1-N/k)$$

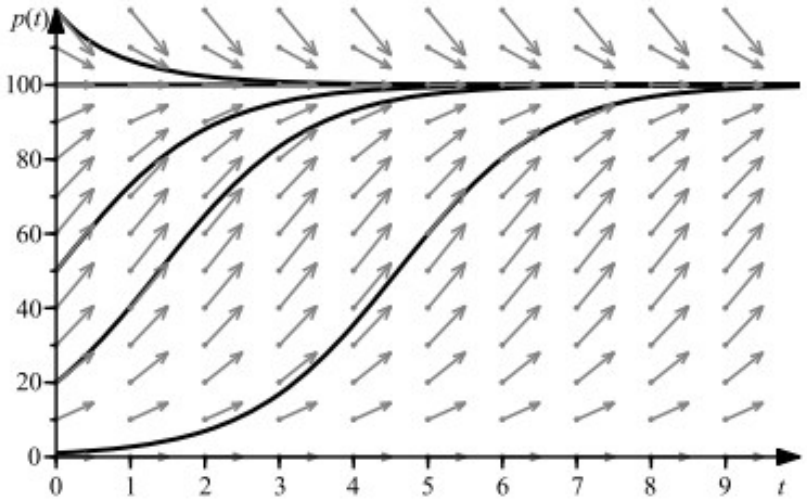
The Verhulst Equation

Solution:

$$N = \frac{k}{1 + Ae^{-rt}}$$



What if the initial population is above carrying capacity?
What if the initial population is 0?



Let's try another few…

What does the graph of the ODE look like? What does the solution look like?

Hint: the dot means "derivative of __ with respect to time (t)"

$$\dot{y} = \cos(t).$$

ODE plot of dy/dt

Solution (y(t))

$$\dot{v} = -k \cdot v,$$

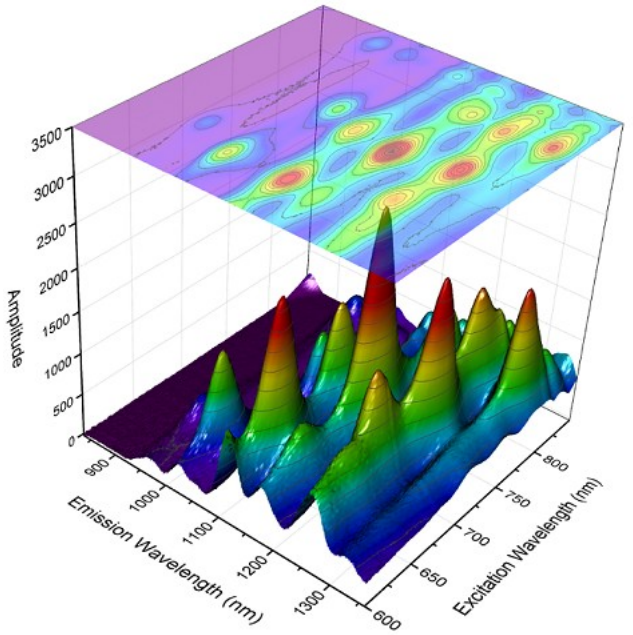
ODE plot of dv/dt

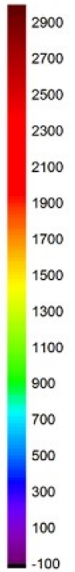
Solution (v(t))

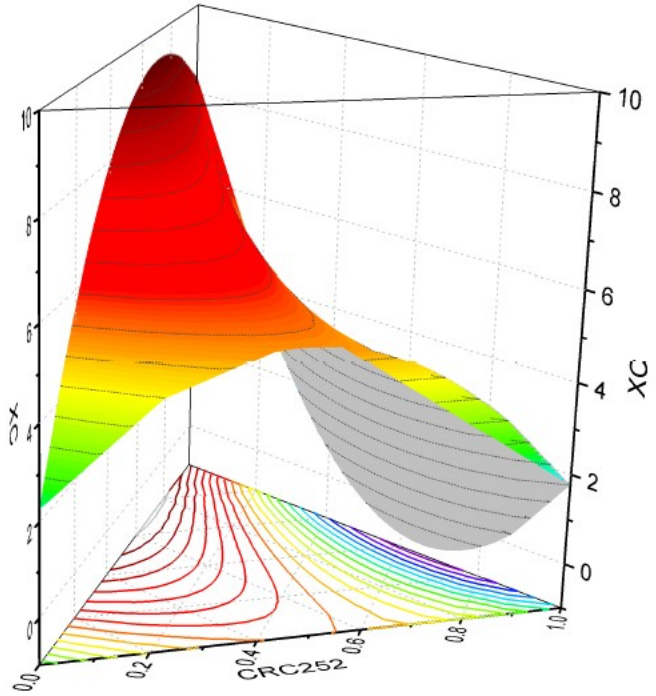
http://lewis500.github.io/diffeq/

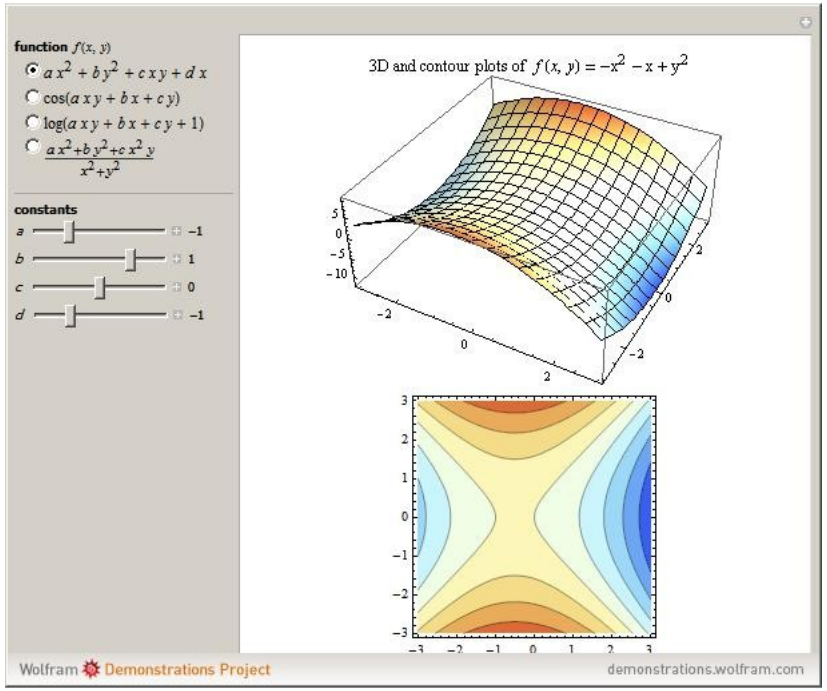
Vector plots: visualizing two dependent variables

Recognize these types of plots? They transform 3D data (two dependent variables + 1 dependent variable) into 2D









That's what a vector plot does.

- If you have differential equations based on two ~independent~ variables X(t) and Y(t), you'll have a systems of equations that might look like this:

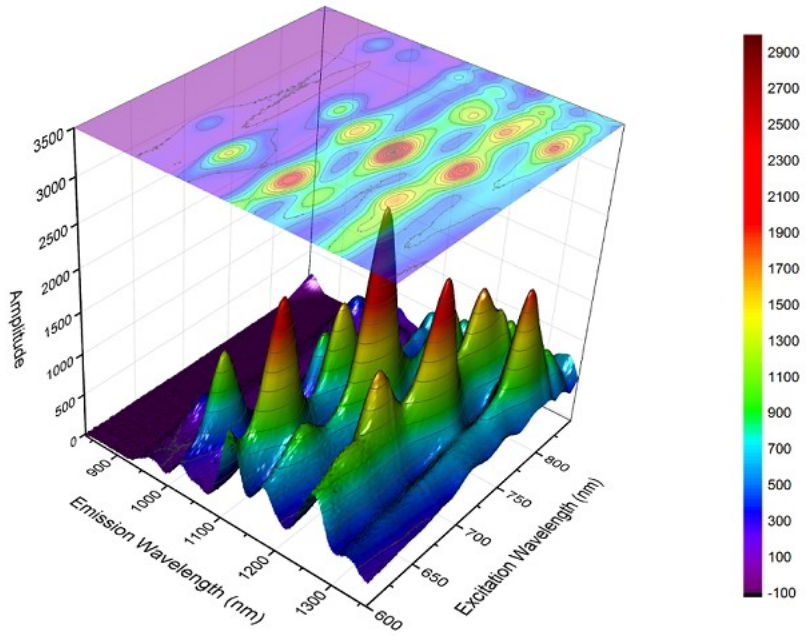
$$\frac{dx}{dt} = ax + by$$
$$\frac{dy}{dt} = cx + dy$$

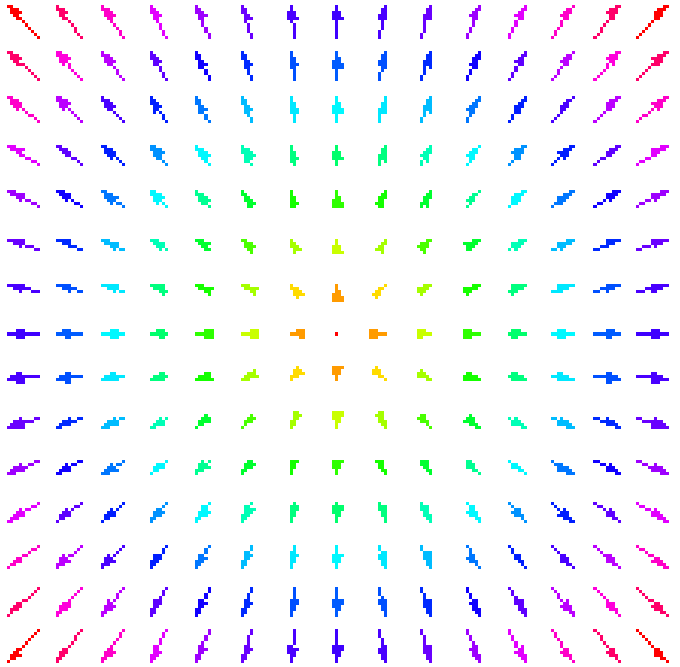
$$\frac{d\vec{\mathbf{x}}}{dt} = A\vec{\mathbf{x}}, \quad \text{where} \quad \vec{\mathbf{x}} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \text{and} \quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Instead of **Z**, this is our dependent variable: remember it is a rate, which is a **VECTOR** not a SCALAR.

X and Y are lumped together here in vector form, but really they are just the two dimensions of our plot (and both are based on t, just like N was in the logistic growth model)

CONTOUR PLOTS VS VECTOR FIELDS





In this contour graph, we have two independent variables (X and Y), and our z axis is the dependent variable, Z(x,y). The color represents the **magnitude**.

When you pick a location (x,y), the graph tells you the value of Z(x,y) at that point.

With vector fields, we have two independent variables (X(t) and Y(t)), and our z axis is the dependent variable, the system of **differential equations.** I.E. a **rate** dependent on X and Y. I.E., each X and Y point will have a **magnitude** (color) AND **direction** (arrow).

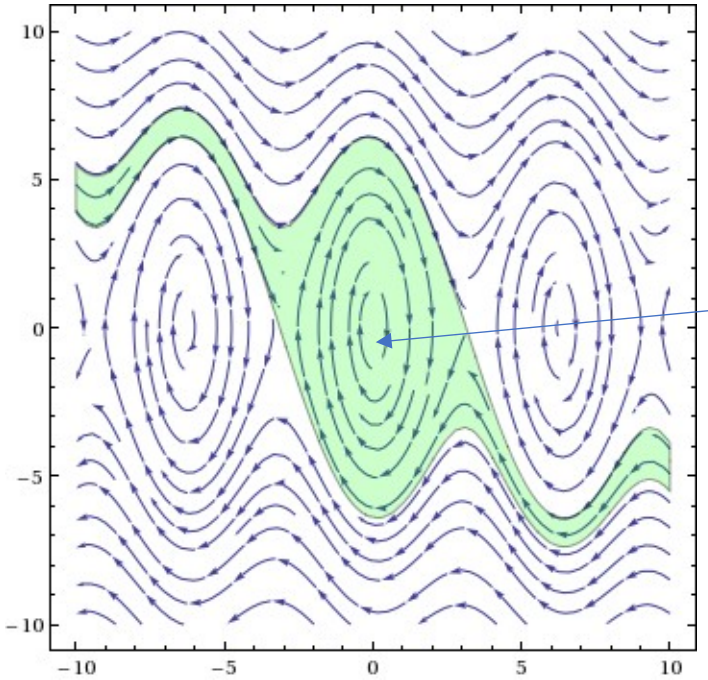
When you pick a location (x,y), the graph tells you which way to move, and how fast. You keep moving until the vector field at your location is 0 (I.E. The rate of change is 0 = no movement).

Grapher Application on Mac

- Examples >> Differential Equations

Why is this important?

- Good way to visualize the dynamics of a system based on where you started (or what the entire space looks like!)
- You can easily see **ATTRACTOR** states, and BASINS of ATTRACTION, by starting from different locations and seeing where they end up.



Attractor state

Anything in the green region gets “sucked in” to the attractor state at the center of the plot.