ODEs → Reactions → Networks

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ODEs (and math in general) are just another language…

- Instead of writing reactions in words that may be ambiguous (interact, etc.), we write them with math that is **precise** and **useful for modeling**
- We know how to read sentences and/or network graphs and turn them into reactions, which we then turn into ODEs
- **REVIEW**: X and Y bind (rate k_b), and upon binding, get converted to Z (rate k_t). All three molecules (X,Y, and Z) get degraded at rates k_x,k_y, and k_z, and synthesized at rates k_1, k_2, and k_3. Write out the ODEs.

We’re going to translate the other way.

- ODEs back to reactions/networks.
- Try these… What do the reactions look like? What about the graph of the derivative and of the **steady state solution**?

$$\frac{dR}{dt} = k_0 + k_1 S - k_2 R,$$

$$\frac{dR_P}{dt} = k_1 S(R_T - R_P) - k_2 R_P.$$

Steady state solution: where the derivative is equal to 0. In other words, the rate of concentration over time is 0, so the concentration isn’t changing, and the system is at a steady state. It’s usually the same as the attractor state we’ve been talking about.

R: response magnitude
S: signal

R_P is the phosphorylated form of the response element (which we suppose to be the active form) R_P = [RP]

$R_T + R_P$ = total concentration of the response element.

A steady-state solution of a differential equation, $dR/dt = f(R)$, is a constant, R_{ss}, that satisfies the algebraic equation $f(R_{ss}) = 0$. In our cases,

$$R_{ss} = \frac{k_0 + k_1 S}{k_2} \tag{a}$$

$$R_{P,ss} = \frac{R_T S}{(k_2/k_1) + S}. \tag{b}$$

More examples

1. Find steady states 2. Sketch the ODE 3. Sketch the solution. 4. Translate back to reactions 5. Make a network out of these reactions. 6. Why did I give each system the name I did?

A: “Sniffer”

$$\frac{dR}{dt} = k_1 S - k_2 X \cdot R$$
$$\frac{dX}{dt} = k_3 S - k_4 X$$

B: “One way switch”

$$\frac{dR}{dt} = k_0 E_P(R) + k_1 S - k_2 X \cdot R$$
$$E_P(R) = G(k_3 R, k_4, J_3, J_4)$$

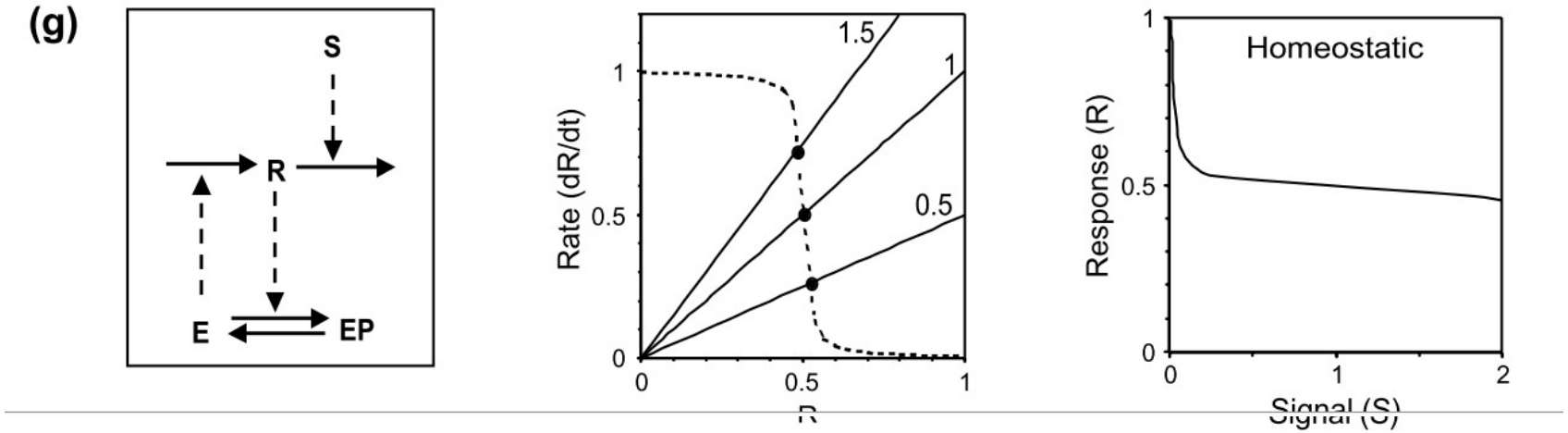
C: “Toggle switch”

$$\frac{dR}{dt} = k_o + k_1 S - k_2 R - k_2' E(R) \cdot R$$
$$E(R) = G(k_3, k_4 R, J_3, J_4)$$

D: “Thermostat”

$$\frac{dR}{dt} = k_0 E(R) - k_2 S \cdot R$$
$$E(R) = G(k_3, k_4 R, J_3, J_4)$$

E(R) is a really gross function…
Assume $E_p(R)$ is an activated form of E(R). Notice that in B, C, and D, the undefined function depends on constants and **only one independent variable**, which is all you need to know!


(g)
S
R
E
EP
1.5
1
0.5
Rate (dR/dt)
0
0.5
1
Homeostatic
Response (R)
0
0.5
1
0
1
2
Signal (S)