Module 4

Lecture 2

Euler's Method

Sarah Groves, Ph.D.

Shayn Peirce-Cottler, Ph.D.



Check-in: what R0 did you get?
What virus family is close to this?

New information! Let's practice pulling from upstream

CONFIDENTIAL OUTBREAK INVESTIGATION
DAY 3 BRIEF

OUTBREAK UPDATE — SEIR MODELING
TO: Industry modeling consultants hired by UVA
FROM: UVA Student Health & Wellness
DATE: February 26, 2026

NEW INFORMATION
Case surveillance now includes daily case counts through day 70.

⚠ Note: The outbreak has not yet peaked in the data provided.

UPDATED EPIDEMIOLOGICAL PARAMETERS

- **Incubation period:** approximately **12–18 days**
- Some students are reporting joint pain as a symptom.
- Early clusters of disease have coincided with midterm exam periods, during which library occupancy reached peak capacity.
- Symptom-based testing criteria were broadened after rash presentation was recognized.

CURRENT TASKS

- Choose **best parameters** for SEIR model that fit the data and disease information using a grid search
- **Predict** the spread of the disease and **when cases will peak**
- Plot the model-predicted infections over time

In **GitHub Desktop:**

- Fetch origin. If it says there are upstream changes, go ahead and pull them!
- If not, choose a branch to merge into main → Choose upstream
- If neither work:
  - Right click on current repository → Open in terminal/command prompt
  - Type git status (make sure you are in the right repo)
  - Type git pull upstream main

If it still isn't working, you may have forgotten to add my repository as upstream.

Next goal: predict peak of epidemic

What do we need to be able to predict the future?

Step 1: Figure out how to solve the SEIR model

Step 2: Fit a model to our data by picking good parameters

Step 3: Extrapolate past the data that we have to see when I(t) peaks in the model

Step 1: Solving the SEIR Model (with a numerical method)

How do we solve ODEs?

1)
Guessing

E.g.: exponential growth/decay

$$y(t) = y_0 e^{kt}$$

$$y'(t) = k y_0 e^{kt} = k y(t)$$

$$y' = ky$$

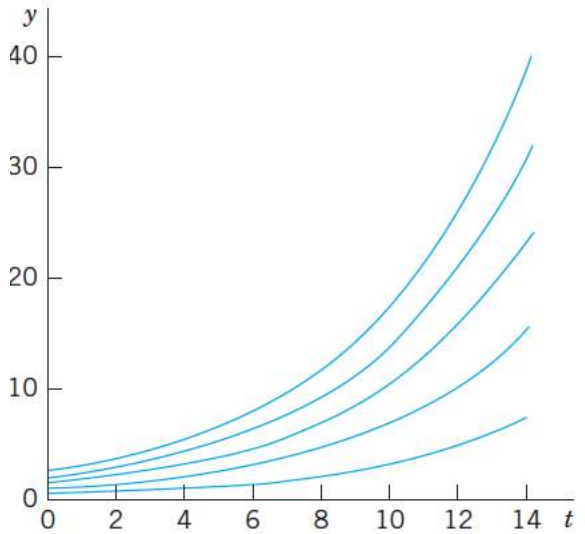


Fig. 4A. Solutions of $y' = 0.2y$ in Example 3 **(exponential growth)**

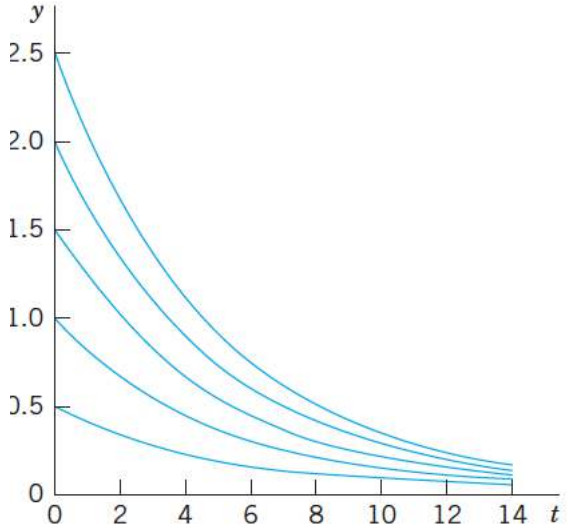


Fig. 4B. Solutions of $y' = -0.2y$ in Example 3 **(exponential decay)**

*Note: I accidentally gave two correct solutions for a problem on your quiz with this

How do we solve ODEs?

2) Separation of variables

$$g(y)\,y' = f(x) \longrightarrow \int g(y)\,y'\,dx = \int f(x)\,dx + c \longrightarrow \int g(y)\,dy = \int f(x)\,dx + c.$$

exampl e

The ODE $y' = 1 + y^2$ is separable because it can be written

$$\frac{dy}{1 + y^2} = dx. \qquad \text{By integration,} \qquad \arctan y = x + c \qquad \text{or} \qquad y = \tan(x + c).$$

How do we solve ODEs?

$$\frac{dy}{dx} = -2x^3 + 12x^2 - 20x + 8.5$$

The initial condition at $x = 0$ is $y = 1$.

3. Numerical methods

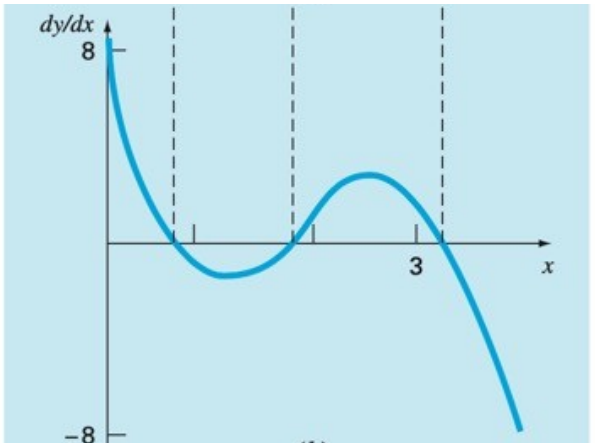
ODE Form: $\frac{dy}{dt} = f(t, y)$

General Form:

$$y_{i+1} = y_i + \phi h$$

$$\phi = f(t_i, y_i)$$

Euler's method



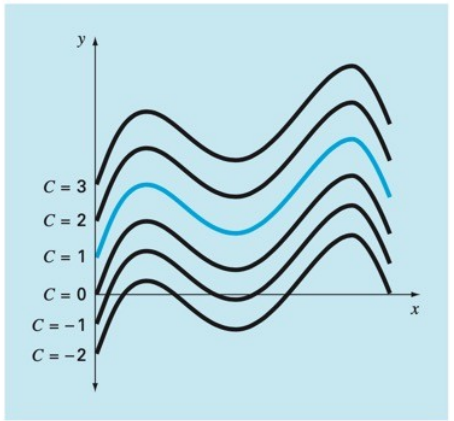
We know:

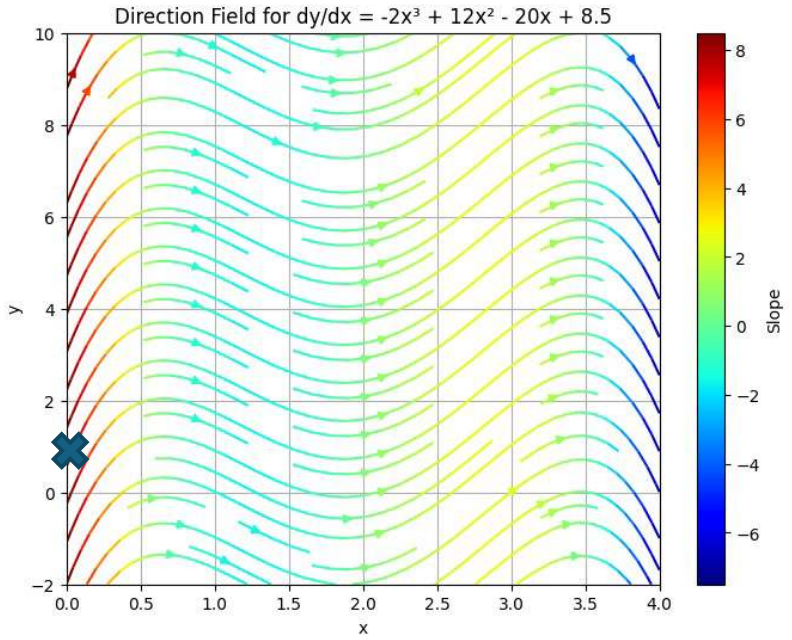
- $y' = f(x)$ at any x, including y_i' and y_{i+1}'
- $y(x_i) = y_i$

We don't know:

- $y(x_{i+1}) = y_{i+1}$

True solution





Solving Systems of ODEs

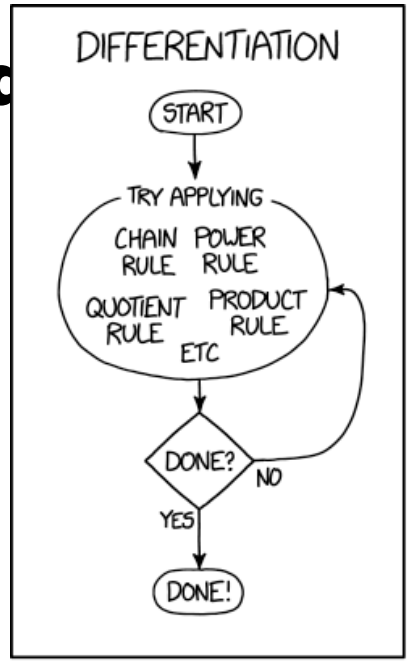
$$\frac{dS}{dt} = -\beta \frac{SI}{N}$$

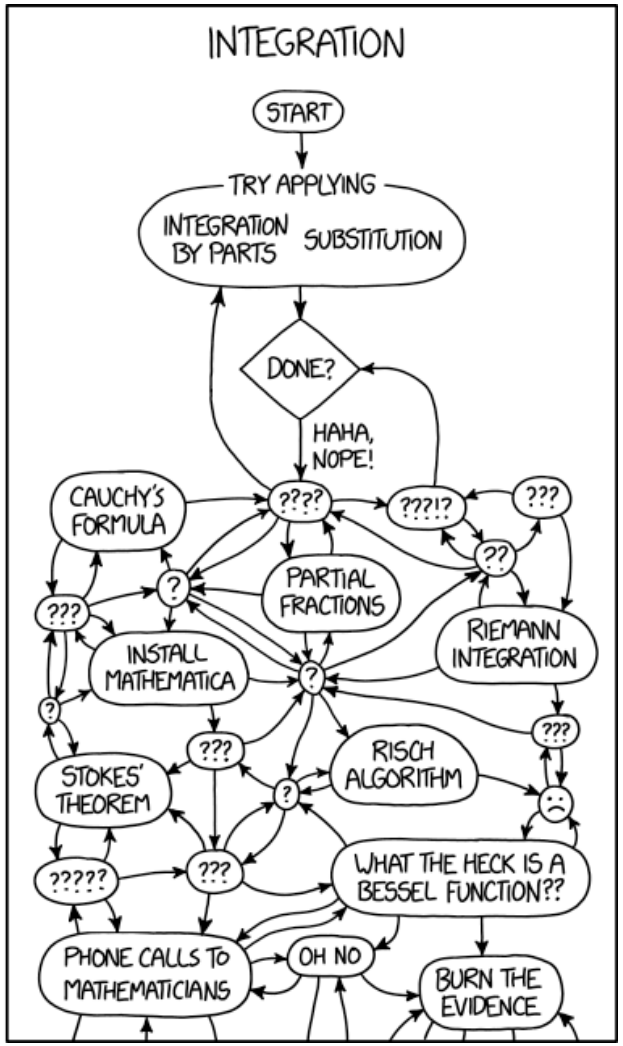
$$\frac{dE}{dt} = \beta \frac{SI}{N} - \sigma E$$

$$\frac{dI}{dt} = \sigma E - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

- Our system is **autonomous and nonlinear**
- Can't solve by guessing
- Can't solve by integration by parts
- CAN approximate numerically with Euler's method





Euler's Method Example

ODE Form: $\frac{dy}{dt} = f(t, y)$

$$\frac{dy}{dx} = \frac{x}{y}$$

I.C. $y(x = -2) = 3$

$h = 1$

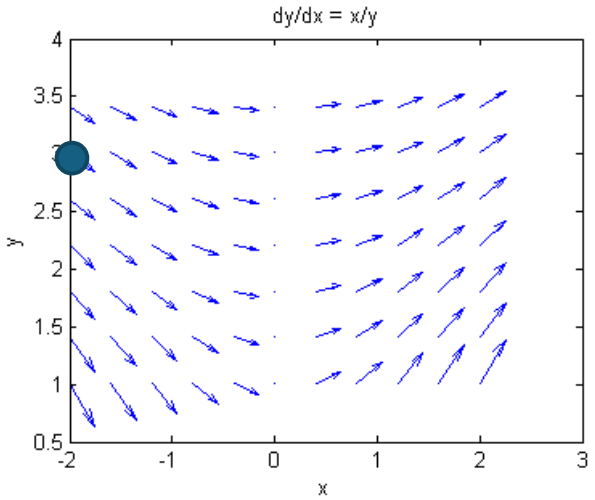
Notice our ODE is dependent on the independent and dependent variables here!

Solve y(x) using Euler's method.

General Form:

$$y_{i+1} = y_i + \phi h$$

$$\phi = f(t_i, y_i)$$



iteration	x_i	y_i	$f(x_i, y_i)$	y_{i+1}
1	-2	3	-2/3	3+(-2/3)*1 = 2 1/3
2	-1	2 1/3	-1/(2 1/3) = -.428	2 1/3 + (-.428)*1
3	0	1.905	0/1.905=0	1.905

Pseudocode for Euler's Method Approximation of SEIR

- INPUTS: beta, sigma, gamma, S0, E0, I0, R0, timepoints, N
- Initialize S, E, I, and R as empty arrays or lists
- Set first item in each list equal to initial values S0, E0, I0, R0
- For each timepoint in timepoints:
  - Calculate the four derivatives at timepoint
  - Calculate S, E, I, and R at timepoint + 1 using Euler's method
- Return S, E, I, and R

Step 2: Fitting the SEIR Model (using optimization)

Fitting the SIR Model Parameters using Euler's Method

$$\frac{dS}{dt} = -\beta \frac{SI}{N}$$

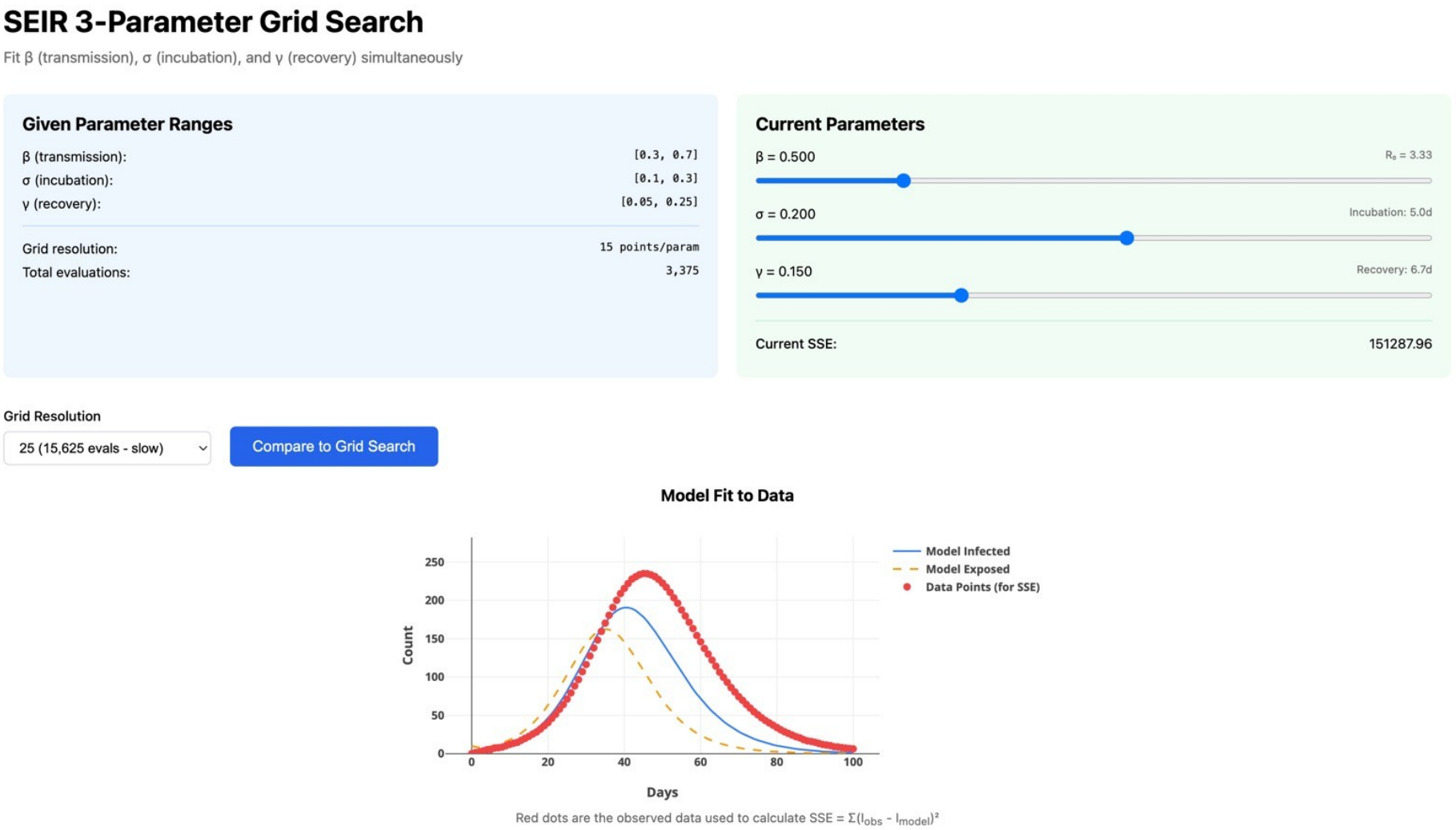
$$\frac{dE}{dt} = \beta \frac{SI}{N} - \sigma E$$

$$\frac{dI}{dt} = \sigma E - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

- Our goal: get S(t), E(t), I(t), and R(t) using Euler's method
- These solutions will depend on our parameters, beta, sigma, and gamma
  - We can choose different values of the parameters to try to match our actual dataset
- In other words, we will **combine Euler's method** with **optimization of SSE** to pick best beta, sigma, and gamma for our dataset

Using the data (red), change the parameters until the model I(t) (blue) matches the data



- With your partner, discuss:
  - How does changing beta affect the model curve? Sigma? Gamma?
  - How can you use the “current SSE” values to tell if you are getting closer to the best fit?
  - Which resolution for the grid search is best?

Grid search optimization pseudocode

- INPUTS: timepoints, N, S0, E0, I0, R0, data
- Initialize a range for beta, sigma, and gamma
- Initialize an empty array of SSE
- Make arrays of values given each range for each parameter
- For b in beta
  - For s in sigma
    - For g in gamma
      - Use the Euler method function you developed to calculate S, E, I, and R given those parameters
      - Calculate the SSE given the model results and the data and append this to the SSE array
- Determine parameters corresponding to lowest SSE
- Return best_beta, best_sigma, and best_gamma and corresponding SSE

HINT: Use R0 you calculated and the info we have to pick a good range!

Step 3: Predict the future

Predicting the peak

- INPUTS: best_beta, best_sigma, best_gamma
- Use the Euler method to run the model out many more days until the data peaks
- How high is the peak? Is that a reasonable value?
- What day will the peak occur?

Next steps in your code

- Implement Euler's method for SEIR modeling
- Plot Euler's method solutions for I(t) and compare to your data
- Guess beta, sigma, and gamma and calculate SSE

HOMEWORK

- DUE TOMORROW at midnight: GitHub Check in #1
  - Includes code from last Thursday and Tuesday (only working on data release #1)
  - If you have new code from today in your repo, don’t worry. We will find the code related to check-in #1
  - Reference .py files in Code folder to ensure you have everything you need
  - You should also have plots, explanations, etc. Tell us your thought process!
- Due March 9 MIDNIGHT (After spring break): GitHub Check in #2
  - Including only code from today’s class